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Econometrics

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Empirical Analysis

**Part 1: Law of Large Number**

*A: N (4, 16)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 4

- Theoretical expectation for the variance is 256

- Theoretical expectation for the standard deviation is 16

*Bullet 3: Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: -4.132694, n = 1: -11.11235, n = 2: 8.416121, n = 3: 6.427478, n = 4: 4.34736,

n = 5: 9.547867, n = 6: 3.059481, n = 7: 3.706787, n = 8: 3.029666, n = 9: 4.319321,

n = 10: 3.975687, n = 11: 3.493304, n = 12: 3.756383, n = 13: 4.002057, n = 14: 4.054314

n = 15: 4.08661, n = 16: 3.96548, n = 17: 4.075769, n = 18: 3.978671, n = 19: 3.994443

n = 20: 4.017804, n = 21: 4.015856, n = 22: 4.016094, n = 23: 4.005732

*Bullet 4:* As the variable n increased, the population size increased by 2^n, the larger the population side the sample mean converges to the actually mean. This is shown on the graph by the points scatter plot representing each value of n and the solid red line representing the actually mean.



*Bullet 5: Parameters of the underlying distribution changed from N (4,16) to N (8,23):*



- Theoretical expectation for the population space is 8

- Theoretical expectation for the variance is 529

- Theoretical expectation for the standard deviation is 23

*Change in n ranging from 0 to 23:*

n = 0: 13.5953, n = 1: 4.637798, n = 2: 8.934698, n = 3: 3.568626, n = 4: 10.45537,

n = 5: 8.117314, n =6: 7.636682, n = 7: 6.951495, n = 8: 8.496338, n = 9: 7.918934

n = 10: 7.123516, n = 11: 7.300564, n = 12: 8.707814, n = 13: 8.092761, n = 14: 8.283422

n = 15: 8.048845, n = 16: 8.112265, n = 17: 8.064176, n = 18: 8.042218, n = 19: 7.980118

n = 20: 7.96545, n = 21: 8.00063, n = 22: 7.978561, n = 23: 7.982067



Changing the parameters of the underlying distribution, did result in the same conclusion as the initial parameters. As the n increases, the sample size increases by 2^n, as the sample sizes converges to infinity or a larger number, the sample mean converges towards the actually mean.

*B: B (50, 0.4)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 20

- Theoretical expectation for the variance is 12

- Theoretical expectation for the standard deviation is 3

*Bullet 3: Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: 17, n = 1: 14, n = 2: 17.75, n = 3: 19.875, n = 4: 19.875, n = 5: 20.21875, n = 6: 20.21875

n = 7: 20.80469, n = 8: 20.20703, n = 9: 20.05078, n = 10: 19.9707, n = 11: 19.89404

n = 12: 19.91138, n = 13: 20.02405, n = 14: 20.01587, n = 15: 20.00482, n = 16: 19.98607

n = 17: 19.99596, n = 18: 19.99721, n = 19: 19.99824, n = 20: 20.00157, n = 21: 19.99677

n = 22: 20.00086, n = 23: 20

*Bullet 4:* As the variable n increased, the population size increased by 2^n, the larger the population side the sample mean converges to the actually mean. This is shown on the graph by the points scatter plot representing each value of n and the solid red line representing the actually mean.



*Bullet 5: Parameters of the underlying distribution changed from B (50,0.4) to B (60,0.2):*



- Theoretical expectation for the population space is 12

- Theoretical expectation for the variance is 9.6

- Theoretical expectation for the standard deviation is 3

*Change in n ranging from 0 to 23:*

n = 0: 11, n = 1: 11.5, n = 2: 12.25, n = 3: 14.875, n = 4: 11.625, n = 5: 12.0625, n = 6: 11.5625

n = 7: 12.57031, n = 8: 12.10156, n = 9: 12.05859, n = 10: 12.07129, n = 11: 11.88086

n = 12: 12.02808, n = 13: 11.97754, n = 14: 12.01306, n = 15: 12.02713, n = 16: 11.98297

n = 17: 11.99447, n = 18: 11.99656, n = 19: 11.99668, n = 20: 11.99517, n = 21: 11.99804

n = 22: 11.99877, n = 23: 11.99859



Changing the parameters of the underlying distribution, did result in the same conclusion as the initial parameters. As the n increases, the sample size increases by 2^n, as the sample sizes converges to infinity or a larger number, the sample mean converges towards the actually mean.

*C: T (10)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 0

- Theoretical expectation for the variance is 1.25

- Theoretical expectation for the standard deviation is 1.118034

*Bullet 3: Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: -0.2975718, n = 1: 0.7586812, n = 2: 0.1474633, n = 3: 0.3381381, n = 4: -0.06997248

n = 5: 0.1719283, n = 6: 0.1413137, n =7: 0.08857669, n = 8: -0.09386723, n = 9: -0.0588841

n = 10: -0.02042506, n = 11: 0.01016093, n = 12: -0.00256291, n =13: -0.01636535

n = 14: -0.00893389, n = 15: 0.003684486, n =16: 0.001317228, n = 17: 0.0006701405

n = 18: 0.001574488, n = 19: 0.001630551, n = 20: 5.336164e-05, n = 21: 0.0008098609

n = 22: 0.0001149164, n = 23: 0.0001948259

*Bullet 4:*



As the variable n increased, the population size increased by 2^n, the larger the population side the sample mean converges to the actually mean. This is shown on the graph by the points scatter plot representing each value of n and the solid red line representing the actually mean.

*Bullet 5: Parameters of the underlying distribution changed from T (10) to T (6):*



- Theoretical expectation for the population space is 0

- Theoretical expectation for the variance is 1.5

- Theoretical expectation for the standard deviation is 1.224745

*Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: -0.2648374, n = 1: -0.2810703, n = 2: 0.7623541, n = 3: -0.09117486, n = 4: 0.2504553

n = 5: -0.1299076, n = 6: -0.1096568, n = 7: 0.01189207, n = 8: -0.001304021

n = 9: -0.06820616, n = 10: -0.02351402, n = 11: 0.06479238, n = 12: 0.02423026

n = 13: -0.01052642, n = 14: 0.00433648, n =15: -0.000496729, n = 16: -0.005883568

n = 17: 0.003068743, n = 18: -0.003454896, n = 19: 0.0004967518, n = 20: -0.0009597099

n = 21: -0.000169593, n = 22: 0.0007762411, n = 23: -0.000106889



Changing the parameters of the underlying distribution, did result in the same conclusion as the initial parameters. As the n increases, the sample size increases by 2^n, as the sample sizes converges to infinity or a larger number, the sample mean converges towards the actually mean.

If the degrees of freedom is not greater than 2 the sample mean will not converge because the theoretical variance would not exist since the equation for theoretical variance of t-distribution is equal to the degrees of freedom divided by the degrees of freedom minus 2. Meaning that if degrees of freedom is equal to 2 the equations will return the degrees of freedom divided by 0 which would result in an error. As well variance must also be greater than 1 so if the degrees of freedom is less than 2, the equation will return a negative number.

*D: F (9, 7)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 1.4

- Theoretical expectation for the variance is 2.032593

- Theoretical expectation for the standard deviation is 1.42569

*Bullet 3: Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: 0.7839077, n = 1: 1.109266, n = 2: 1.652723, n = 3: 1.33238, n = 4: 0.8935576

n = 5: 1.352624, n = 6: 1.815242, n = 7: 1.383447, n = 8: 1.29338, n = 9: 1.367039

n = 10: 1.411516, n = 11: 1.371937, n = 12: 1.426853, n = 13: 1.373101, n =14: 1.396752

n = 15: 1.391425, n = 16: 1.399502, n = 17: 1.407535, n =18: 1.39847, n = 19: 1.397793

n = 20: 1.399504, n = 21: 1.397596, n = 22: 1.399085, n = 23: 1.399445

*Bullet 4:*



As the variable n increased, the population size increased by 2^n, the larger the population side the sample mean converges to the actually mean. This is shown on the graph by the points scatter plot representing each value of n and the solid red line representing the actually mean.

*Bullet 5: Parameters of the underlying distribution changed from F (9, 7) to T (14, 6):*



- Theoretical expectation for the population space is 1.5

- Theoretical expectation for the variance is 2.892857

- Theoretical expectation for the standard deviation is 1.70084

*Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: 0.782203, n = 1: 2.316343, n = 2: 1.644827, n = 3: 1.706634, n = 4: 1.361146

n = 5: 1.188282, n = 6: 1.782271, n = 7: 1.388135, n = 8: 1.564025, n = 9: 1.665038

n = 10: 1.475946, n = 11: 1.467491, n = 12: 1.502838, n = 13: 1.509434, n = 14: 1.509643

n = 15: 1.491034, n = 16: 1.491373, n = 17: 1.502952, n = 18: 1.496739, n = 19: 1.496301

n = 20: 1.502105, n = 21: 1.499488, n = 22: 1.500663, n = 23: 1.500092

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Changing the parameters of the underlying distribution, did result in the same conclusion as the initial parameters. As the n increases, the sample size increases by 2^n, as the sample sizes converges to infinity or a larger number, the sample mean converges towards the actually mean.

The second degrees of freedom parameter in the F-Distribution has to be greater than 4 in order for the theoretical variance to exist. Since a df2 of 4 would result in the equation for the theoretical variance equal to 0 which would result in an error.

**Part 2: Central Limit Theorem**

*A: N (20, 25)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 20

- Theoretical expectation for the variance is 625

- Theoretical expectation for the standard deviation is 25

*Bullet 3:*

*-Large n case (where n = 100000) Sample Means:*

[1] 20.01548, 19.98959, 19.97940, 19.94889… [101] 20.04712, 19.91620, 19.91964, 19.90218 … [201] 20.09528, 20.03387, 19.90140, 19.95266… [301] 19.86299, 20.02776, 19.95074, 20.03371… [401] 19.93794, 20.24957, 19.99904, 19.92565… [501] 20.07289, 20.09073, 20.05699, 20.06221… [601] 19.98266, 20.12935, 19.99545, 20.05965… [701] 19.92815, 20.04783, 20.05169, 20.10630… [801] 20.00442, 19.93581, 19.97710, 19.97610

… [901] 20.03827, 19.99171, 20.05753, 19.88416…

*- Small n case (where n = 10) Sample Means:*

[1] 29.71390353, 29.47248662, 5.30619361, 20.35467601… [150] 19.27618260, 22.47102101, -0.04104633, 18.91764415… [300] 13.77398122, 9.43186677, 23.08459097, 29.65909941… [450] 27.69768239, 17.77666251, 16.94359358, 12.42153047… [600] 25.70775159, 27.00711521, 26.33223806, 14.87710712… [750] 18.97185940, 28.78738243, 20.32326663, 27.59390275… [900] 12.90951230, 20.28079414, 23.00021153, 10.44154102, 15.10860340…

*Bullet 4: (Sample means with sample size 10)*



This graph shows that even though the samples are drawn from a large population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 5: (Sample means with sample size 100000)*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size.

*Bullet 6: (Normalized sample mean (size 10) with normal distribution):*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 7: (Normalized sample mean (size 100000) with normal distribution):*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 8: Parameters of the underlying distribution changed from N (20,25) to N (30, 6):*



- Theoretical expectation for the population space is 30

- Theoretical expectation for the variance is 36

- Theoretical expectation for the standard deviation is 6

*Large n case (where n = 100000) Sample Means:*

[1] 29.98921, 29.99691, 30.02541, 30.00153… [100] 30.02920, 30.01588, 30.01913, 29.99240… [200] 29.99407, 29.96501, 30.00562, 29.99238… [300] 30.02440, 29.95343, 30.03493, 29.99889… [400] 30.02124, 29.98687, 29.99432, 30.01618… [500] 29.99281, 29.98385, 30.01062, 29.99898…[600] 30.00001, 30.03401, 30.00305, 30.03848… [700] 30.01876, 30.00251, 30.01369, 30.00854… [800] 30.00327, 29.98853, 30.00002, 29.99258… [900] 30.00767, 29.99263, 30.00411, 29.99629…

*Small n case (where n = 10) Sample Means:*

[1] 32.70029, 29.00809, 30.29667, 27.72423… [150] 33.36675, 31.55160, 30.29983,

25.78260… [300] 30.80959, 29.93874, 32.71586, 30.50761… [450] 32.12323, 27.51090, 29.27001, 31.74968… [600] 29.71853, 26.52864, 29.28666, 27.81614… [750] 29.11715, 29.01691, 30.89513, 30.89859… [900] 30.06128, 31.15953, 29.86930, 31.05715…

*(Sample means with sample size 10) Plot:*



This graph shows that even though the samples are drawn from a large population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Sample means with sample size 100000) Plot:*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size.

*(Normalized sample mean (size 10) with normal distribution) Plot:*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*(Normalized sample mean (size 100000) with normal distribution) Plot:*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

The change in parameters for Normal Distribution did result in the same conclusion as the parameters N (20,25), the independent sample means are added their normalized sum converges towards a normal distribution as the number of samples means goes towards infinity.

*B: B (40, 0.2)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 8

- Theoretical expectation for the variance is 6.4

- Theoretical expectation for the standard deviation is 2.529822

*Bullet 3:*

*-Large n case (where n = 100000) Sample Means:*

[1] 7.98899, 7.98726, 8.00745, 7.99523… [100] 7.98571, 8.01168, 8.00227, 8.00870… [200] 8.00774, 7.99829, 7.99330, 7.99976…. [300] 8.00312, 7.99642, 8.00138, 8.00307… [400] 8.00143, 8.01806, 8.01209, 8.00489… [500] 7.99436, 8.00601, 7.99668, 8.00657… [600] 7.99675, 7.99305, 7.99257, 8.01241… [700] 8.01292, 7.99201, 8.00133, 8.00342… [800]

8.01189, 7.99429, 7.99950, 7.99480… [900] 7.98709, 8.01175, 8.01764, 7.99286…

*- Small n case (where n = 10) Sample Means:*

[1] 8.5, 8.0, 11.0, 7.5… [150] 9.5, 10.0, 10.5, 6.5… [300] 6.5, 7.0, 7.0, 7.5… [450] 7.5, 9.0, 9.0, 9.0… [600] 12.5, 5.0, 11.0, 6.5… [750] 8.0, 11.0, 8.0, 6.0… [900] 12.0, 10.0, 8.5, 6.5…

*Bullet 4: (Sample means with sample size 10)*

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This graph shows that even though the samples are drawn from a large population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 5: (Sample means with sample size 100000)*



This graph shows that even though the samples are drawn from a large population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 6: (Normalized sample mean (size 10) with normal distribution):*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 7: (Normalized sample mean (size 100000) with normal distribution):*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 8: Parameters of the underlying distribution changed from B (40, 0.2) to N (60, 0.2):*



- Theoretical expectation for the population space is 12

- Theoretical expectation for the variance is 9.6

- Theoretical expectation for the standard deviation is 3.098387

*Large n case (where n = 100000) Sample Means:*

[1] 12.00534 12.00652 12.00316 12.00374… [100] 11.99882, 11.99975, 12.00776, 11.98413… [200] 11.98311, 11.99903, 11.98480, 11.98838… [300] 11.98218, 12.01841, 11.99878, 11.98457… [400] 12.00772, 12.00982, 12.00347, 11.99718… [500] 12.00346, 12.00539, 12.00809, 12.00322… [600] 12.00059, 11.98887, 12.00681, 11.99558… [700] 12.01406, 12.01133, 12.00577, 11.99474… [800] 11.98856, 11.99321, 12.00467, 12.01090… [900] 12.00923, 11.99473, 12.01511, 12.01863…

*Small n case (where n = 10) Sample Means:*

[1] 12.0 13.0 14.0 14.5… [150] 13.0, 13.0, 14.0, 11.0… [300] 16.0, 12.5, 13.0, 10.0… [450] 12.5, 13.5, 12.0, 8.5… [600] 16.0, 11.0, 10.0, 13.5… [750] 13.5, 9.5, 10.5, 13.0… [900] 14.0, 14.5, 11.0, 12.0…

*(Sample means with sample size 10) Plot:*



This graph shows that even though the samples are drawn from a large population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Sample means with sample size 100000) Plot:*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Normalized sample mean (size 10) with normal distribution) Plot:*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*(Normalized sample mean (size 100000) with normal distribution) Plot:*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

Changing the initial parameters from B (40, 0.2) to B (60, 0.2) did result in the same conclusion as the initial parameters. Regardless of actually sample size as long as there are sufficient trials ran the normalized sum of the sample mean still converges towards normal distribution or a bell shaped graph, as the number of trials ran for the sample mean approaches infinity.

*C: T (10)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 0

- Theoretical expectation for the variance is 1.25

- Theoretical expectation for the standard deviation is 1.118034

*Bullet 3:*

*-Large n case (where n = 100000) Sample Means:*

[1] 1.6189144991, -0.3482714056, 0.2169416184, -1.2636915249… [100] 1.1387057310,

-1.0439225248, 0.1641520852, -1.6300150453… [200] -1.4552010669, -1.0260351362,

-0.7739332882, -0.4115385798… [300] -0.6870822303, 0.5350743090, -0.7564707254, 0.9438709549… [400] -0.5784697504, -0.2155607233, 0.8292611645, 0.4313122073… [500] 0.2928814727, 0.6245627314, -0.2137876844, -0.2987927752… [600] 0.3047832363,

-0.0482868400, -0.3021756351, 1.0317814550, -1.0710806589… [700] -0.1195161882, 0.1346184920, 0.2913480067, 1.3763737467… [800] 0.3499325669, -0.1883059501,

-0.1894727701, -0.8925895620… [900] -0.1988167881, -0.1824933925, -0.3621569686, 0.1319900284…

*Small n case (where n = 10) Sample Means:*

[1] 3.305360e-03, 1.035409e-03, 5.147782e-04, 1.496170e-03… [150] -3.614020e-03,

-2.775377e-03, -6.481389e-03, -6.104185e-03… [300] -2.153086e-03, 2.673285e-03,

-5.890258e-04, -2.778565e-03… [450] 1.797212e-03, 1.708669e-03, 1.606016e-05,

-1.920905e-03… [600] 7.771304e-03, -8.505457e-04, 3.932398e-03, -1.600786e-03… [750] 3.993838e-03, -6.905112e-03, 2.877488e-03, -3.931815e-03… [900] 1.829152e-03,

4.024206e-03, -2.435237e-03, -2.092415e-04…

*Bullet 4: (Sample means with sample size 10)*



This graph shows that even though the samples are drawn from a large population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 5: (Sample means with sample size 100000)*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 6: (Normalized sample mean (size 10) with normal distribution):*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 7: (Normalized sample mean (size 100000) with normal distribution):*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 8: Parameters of the underlying distribution changed from T (10) to T (4):*



- Theoretical expectation for the population space is 0

- Theoretical expectation for the variance is 2

- Theoretical expectation for the standard deviation is 1.414214

*Large n case (where n = 100000) Sample Means:*

[1] 0.4270709831, 0.0277202141, 0.1364651016, 0.1013249563… [100]-0.7796637249*,* 0.6882642825, 1.5005906798, -0.0604352126… [200] 0.1180835836*,* 0.0091976752,

-0.9295486841, -1.1162103318… [300] 2.7701376735, -0.8535874973, 0.3952773317, 0.8571275642… [400] 0.0710569272, -0.1387883755, 0.8137777409, 0.3984778742… [500]

-0.1569605462, 0.7347782231, 0.1414485021, 1.6937212307… [600] 0.8711299408*,*

-0.2814471944, 1.1949933038, -0.5711849425… [700] 0.5815360187, 0.8705616342, 0.0721511338, -1.1091004831… [800] -0.5611913391, 0.5746496049, -0.5657490146,

-0.0005795055… [900] -0.0617739403, 0.2041133614, 0.4390688287, -1.1677774135…

*Small n case (where n = 10) Sample Means:*

[1] 2.037760e-03, -1.010079e-03, -3.943408e-03, -3.621258e-03… [150] 5.631026e-03,

-1.195721e-04, -3.552904e-03, -1.990321e-03… [300] -5.122902e-03, -7.092626e-03,

1.822270e-03, -1.162404e-06… [450] -7.035170e-03, -3.182498e-05, -5.651296e-03,

-1.266691e-03… [600] 5.707195e-03, -1.288401e-03, 1.043061e-02, -3.181320e-03… [750]

-7.789713e-04, 3.656535e-03, 2.306427e-03, 4.385914e-03… [900] 2.346061e-03,

5.715224e-03, 1.859380e-03, 4.779841e-04…

*(Sample means with sample size 10) Plot:*



This graph shows that even though the samples are drawn from a small population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Sample means with sample size 100000) Plot:*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Normalized sample mean (size 10) with normal distribution) Plot:*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*(Normalized sample mean (size 100000) with normal distribution) Plot:*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

Changing the initial parameters did result in the same conclusion, the sample means when normalized converged to normal distribution or had a bell-shaped graph. The degrees of freedom had to be greater than 2 in order for a theoretical variance to exist, if the degrees of freedom were less than two the code would result in an order since the equation for variance would have a numerator being divided by 0.

*D: F (8, 6)*

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 1.5

- Theoretical expectation for the variance is 3.375

- Theoretical expectation for the standard deviation is 1.837117

*Bullet 3:*

*-Large n case (where n = 100000) Sample Means:*

[1] 1.489968 1.492438 1.505283 1.507284… [100] 1.508430 1.501950 1.496105 1.502626… [200] 1.504250, 1.495650, 1.503287, 1.501418… [300] 1.491390, 1.499190, 1.496914, 1.489444… [400] 1.503180, 1.515587, 1.503048, 1.501583… [500] 1.497107, 1.505349, 1.496037, 1.502100… [600] 1.499253, 1.487974, 1.503570, 1.496704… [700] 1.492669, 1.495692, 1.489604, 1.513637… [800] 1.496584, 1.494521, 1.494779, 1.501900… [900] 1.501858, 1.491989, 1.501823, 1.505192…

*Small n case (where n = 10) Sample Means:*

[1] 0.7885366, 0.7276133, 1.7211586, 1.3460152… [150] 0.4237649, 0.6381685, 0.7403645, 2.7673580… [300] 0.6391700, 2.3459699, 1.6964236, 2.2073751… [450] 1.1494122, 1.5878862, 1.0398289, 1.0139282… [600] 0.3739543, 1.3460093, 0.6782652, 0.8668595… [750] 0.7382300, 0.8459532, 1.7298714, 0.4080057… [900] 1.2987620, 1.3072711, 1.4890003, 1.2103312…

*Bullet 4: (Sample means with sample size 10)*



This graph shows that even though the samples are drawn from a small population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 5: (Sample means with sample size 100000)*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 6: (Normalized sample mean (size 10) with normal distribution):*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 7: (Normalized sample mean (size 100000) with normal distribution):*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 8: Parameters of the underlying distribution changed from F (8, 6) to F (10, 5):*



- Theoretical expectation for the population space is 1.666667

- Theoretical expectation for the variance is 7.222222

- Theoretical expectation for the standard deviation is 2.687419

*Large n case (where n = 100000) Sample Means:*

[1] 1.666256, 1.658849, 1.646969, 1.661555… [100] 1.665501, 1.670490, 1.646305, 1.666960… [200] 1.668710, 1.665996, 1.676396, 1.656870… [300] 1.664826, 1.668987, 1.663712, 1.665884… [400] 1.673445, 1.670923, 1.661185, 1.669252… [500] 1.649942, 1.657695, 1.663161, 1.664216… [600] 1.658416, 1.658787, 1.673708, 1.652412… [700] 1.667489, 1.669590, 1.654338, 1.674353… [800] 1.669782, 1.659227, 1.659755, 1.660704… [900] 1.664752, 1.678156, 1.655193, 1.652027…

*Small n case (where n = 10) Sample Means:*

[1] 2.6908372, 1.8869150, 2.5981422, 1.0225909… [150] 0.5025644, 0.6474854, 1.2127150, 0.4402552… [300] 0.4811877, 3.0183309, 0.8064196, 1.9670249… [450] 3.3167087, 3.4085518, 2.0770549, 1.7924061… [600] 0.7460278, 1.5789291, 2.7880184, 1.4604535… [750] 1.5166933, 1.1885381, 1.6442673, 0.9154946… [900] 1.9945219, 1.6819804, 2.9415047, 1.4809845…

*(Sample means with sample size 10) Plot:*



This graph shows that even though the samples are drawn from a small population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Sample means with sample size 100000) Plot:*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Normalized sample mean (size 10) with normal distribution) Plot:*

This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*(Normalized sample mean (size 100000) with normal distribution) Plot:*



This graph shows that the independent large sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

Changing the initial parameters did result in the same conclusion. The distribution followed a normal distribution when the sample means were added, and the normalized sum of the samples had a bell-shaped graph as the number of trials of samples converged towards infinity regardless of the actual size of the sample. The second degrees of freedom must be large than 4 because if it is not, the theoretical variance does not exist. Since the equation for a theoretical variance would result in a denominator of 0.

**EXTRA CREDIT:**  
*Law of Law Numbers: Chi Square Distribution:  
Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 5

- Theoretical expectation for the variance is 10

- Theoretical expectation for the standard deviation is 3

*Bullet 3: Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: 10.73779, n = 1: 12.0802, n = 2: 10.8997, n = 3: 7.755688, n = 4: 9.506668

n = 5: 8.132107, n = 6: 6.80647, n = 7: 8.185725, n = 8: 8.097908, n = 9: 7.848159

n = 10: 7.984825, n = 11: 8.118295, n = 12: 8.081494, n = 13: 7.98264, n = 14: 8.03581

n = 15: 8.017813, n = 16: 7.984002, n = 17: 7.998986, n = 18: 8.004964, n = 19: 8.000463

n = 20: 8.002536, n = 21: 7.996272, n = 22: 8.000135, n = 23: 7.998162

*Bullet 4:*

As the variable n increased, the population size increased by 2^n, the larger the population side the sample mean converges to the actually mean. This is shown on the graph by the points scatter plot representing each value of n and the solid red line representing the actually mean.

**

*Bullet 5: Parameters of the underlying distribution changed from Chi (5) to Chi (8):*



- Theoretical expectation for the population space is 8

- Theoretical expectation for the variance is 16

- Theoretical expectation for the standard deviation is 4

*Sample mean for each value ‘n’ ranging from 0 to 23:*

n = 0: 11.16881, n = 1: 8.878464, n = 2: 8.467649, n = 3: 7.90207, n = 4: 7.898659

n = 5: 8.095928, n = 6: 9.044472, n = 7: 7.717692, n = 8: 8.386913, n = 9: 7.814677

n = 10: 8.035446, n = 11: 8.134589, n = 12: 7.914202, n = 13: 8.06777, n = 14: 7.991156

n = 15: 7.977519, n = 16: 8.029417, n = 17: 7.995013, n = 18: 7.993677, n = 19: 7.99387

n = 20: 7.998565, n = 21: 7.994166, n = 22: 7.997617, n = 23: 7.99981



Changing the parameters of the underlying distribution, did result in the same conclusion as the initial parameters. As the n increases, the sample size increases by 2^n, as the sample sizes converges to infinity or a larger number, the sample mean converges towards the actually mean.

**Central Limit Theorem: Chi Square Distribution:**

*Bullet 1:*



*Bullet 2:*

- Theoretical expectation for the population space is 5

- Theoretical expectation for the variance is 10

- Theoretical expectation for the standard deviation is 3.162278

*Bullet 3:*

*Large n case (where n = 100000) Sample Means:*

[1] 4.99820, 1 4.990330, 5.006359, 5.006325, 5.008424… [100] 5.004398, 5.004898, 4.990210, 5.006703, 5.003125… [200] 4.996682, 5.008257, 4.998135, 4.987201, 4.998709… [300] 4.997916, 5.002419, 5.011851, 4.988579, 4.997849… [400] 4.999909, 4.999257, 5.026635, 5.001490, 4.986556… [500] 5.010118, 4.999768, 4.998989, 5.006099, 5.000480… [600] 4.996712, 5.001905, 5.005400, 4.995800, 4.992007… [700] 4.986205, 5.011634, 4.982637, 5.004765, 5.001341… [800] 4.997176, 4.992011, 4.981162, 5.006749, 4.985285… [900] 5.005815, 5.001207, 5.007605, 4.998199, 5.001990…

*Small n case (where n = 10) Sample Means:*

[1] 5.2860942, 8.4532151, 8.3719858, 4.7914230… [150] 6.6696341, 2.3363729, 2.9510682, 5.0648147… [300] 10.6285939, 3.9508195, 9.9443749, 3.6156203… [450] 4.5652972, 3.6520066, 4.4687421, 3.7150767… [600] 6.3707507, 2.7544189, 3.9994972, 6.2835436… [750] 6.0361080, 2.7885716, 3.8437001, 1.9665888… [900] 3.3306461, 3.8456377, 5.0210116, 6.8986510…

*Bullet 4: (Sample means with sample size 10)*



This graph shows that even though the samples are drawn from a small population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 5: (Sample means with sample size 100000)*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 6: (Normalized sample mean (size 10) with normal distribution):*



This graph shows that the independent small sample means drawn from the population are added, they normalized sum converges towards a normal distribution as the amount of sample means increases towards infinity. This happens regardless of the sample size as long as it is repeated a large amount of times.

*Bullet 7: (Normalized sample mean (size 100000) with normal distribution):*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*Bullet 8: Parameters of the underlying distribution changed from Chi (5) to Chi (10):*



- Theoretical expectation for the population space is 10

- Theoretical expectation for the variance is 20

- Theoretical expectation for the standard deviation is 4.472136

*Large n case (where n = 100000) Sample Means:*

[1] 10.010009, 9.997003, 10.013483, 10.013492… [100] 9.996825, 9.991176, 10.000862, 9.991005… [200] 10.003366, 10.017139, 9.988096, 9.989592… [300] 10.000037, 9.989674, 10.005918, 10.009459… [400] 9.983758, 10.005360, 9.994023, 10.003034… [500] 10.002692, 10.006771, 10.020844, 9.980499… [600] 9.993792, 9.995204, 10.008711, 9.980748… [700] 10.004238, 10.020925, 9.983888, 10.014409… [800] 10.022008, 9.992514, 10.011174, 9.989478… [900] 10.001080, 10.006381, 10.036537, 9.947679…

*Small n case (where n = 10) Sample Means:*

[1] 12.166184, 12.217840, 10.393928, 8.929500… [150] 13.628428, 8.679798, 5.863517, 10.181211… [300] 10.453109, 5.064511, 10.797992, 7.785304… [450] 5.789025, 9.083125, 6.061710, 9.593918… [600] 8.434292, 10.215066, 10.733476, 3.164751… [750] 6.894107, 6.626381, 8.467848, 17.722700… [900] 11.496653, 7.990125, 6.293620, 9.198118…

*(Sample means with sample size 10)*



This graph shows that even though the samples are drawn from a small population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Sample means with sample size 100000)*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Normalized sample mean (size 10) with normal distribution):*



This graph shows that even though the samples are drawn from a small population and the sample size is small the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.

*(Normalized sample mean (size 100000) with normal distribution):*



This graph shows that even though the samples are drawn from a large population and the sample size is large the distribution still closely approximated by normal distribution. Meaning that since the process was performed enough times, there is enough data to show that the distribution resembles closely to that of normal distribution regardless of the same size. This happens regardless of the sample size following normal distribution.